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## Measurement of Central Tendency

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### Central Tendency

In statistics, a central tendency (or, more commonly, a measure of central tendency) is a central value or a typical value for a probability distribution. It is occasionally called an average or just the centre of the distribution. The most common measures of central tendency are the arithmetic mean, the median and the mode. A central tendency can be calculated for either a finite set of values or for a theoretical distribution, such as the normal distribution. Occasionally authors use central tendency (or centrality) to mean “the tendency of quantitative data to cluster around some central value”. This meaning might be expected from the usual dictionary definitions of the words tendency and centrality. Those authors may judge whether data has a strong or a weak central tendency based on the statistical dispersion, as measured by the standard deviation or something similar.

The term “central tendency” dates from the late 1920s.

### Measures of Central Tendency

The following may be applied to one-dimensional data. Depending on the circumstances, it may be appropriate to transform the data before calculating a central tendency. Examples are squaring the values or taking logarithms. Whether a transformation is appropriate and what it should be depend heavily on the data being analyzed.

- ❖ Arithmetic mean (or simply, mean) – the sum of all measurements divided by the number of observations in the data set
- ❖ Median – the middle value that separates the higher half from the lower half of the data set. The median and the mode are the only measures of central tendency that can be used for ordinal data, in which values are ranked relative to each other but are not measured absolutely.
- ❖ Mode – the most frequent value in the data set. This is the only

central tendency measure that can be used with nominal data, which have purely qualitative category assignments.

- ❖ Geometric mean – the  $n$ th root of the product of the data values, where there are  $n$  of these. This measure is valid only for data that are measured absolutely on a strictly positive scale.
- ❖ Harmonic mean – the reciprocal of the arithmetic mean of the reciprocals of the data values. This measure too is valid only for data that are measured absolutely on a strictly positive scale.
- ❖ Weighted mean – an arithmetic mean that incorporates weighting to certain data elements
- ❖ Truncated mean – the arithmetic mean of data values after a certain number or proportion of the highest and lowest data values have been discarded.
  - o Interquartile mean (a type of truncated mean)
- ❖ Midrange – the arithmetic mean of the maximum and minimum values of a data set.
- ❖ Midhinge – the arithmetic mean of the two quartiles.
- ❖ Trimean – the weighted arithmetic mean of the median and two quartiles.
- ❖ Winsorized mean – an arithmetic mean in which extreme values are replaced by values closer to the median.

Any of the above may be applied to each dimension of multi-dimensional data, but the results may not be invariant to rotations of the multi-dimensional space. In addition, there is the

- ❖ Geometric median - which minimizes the sum of distances to the data points. This is the same as the median when applied to one-dimensional data, but it is not the same as taking the median of each dimension independently. It is not invariant to different rescaling of the different dimensions.

### **Solutions to Variational Problems**

Several measures of central tendency can be characterized as solving a variational problem, in the sense of the calculus of variations, namely minimizing variation from the centre. That is, given a measure of statistical dispersion, one asks for a measure of central tendency that minimizes variation: such that variation from the centre is minimal among all choices of centre. In a quip, “dispersion precedes location”. In the sense of  $L^p$  spaces, the correspondence is:

$L^p$	<i>dispersion</i>	<i>central tendency</i>
$L^1$	average absolute deviation	median
$L^2$	standard deviation	mean
$L^\infty$	maximum deviation	midrange

Thus standard deviation about the mean is lower than standard deviation about any other point, and the maximum deviation about the